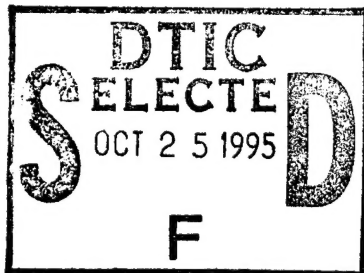TECHNICAL REPORT ARCCB-TR-95028

CONVERGENCE OF NUMERICAL BOX-COUNTING AND CORRELATION INTEGRAL MULTIFRACTAL ANALYSIS TECHNIQUES

L.V. MEISEL
M.A. JOHNSON

APRIL 1995

US ARMY ARMAMENT RESEARCH,
DEVELOPMENT AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
BENÉT LABORATORIES
WATERVLIET, N.Y. 12189-4050

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REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE April 1995	3. REPORT TYPE AND DATES COVERED Final

4. TITLE AND SUBTITLE
CONVERGENCE OF NUMERICAL BOX-COUNTING AND CORRELATION INTEGRAL MULTIFRACTAL ANALYSIS TECHNIQUES

5. FUNDING NUMBERS
AMCMS: 6111.02.H611.1

6. AUTHOR(S)

L.V. Meisel and M.A. Johnson

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

U.S. Army ARDEC
Benét Laboratories, AMSTA-AR-CCB-O
Watervliet, NY 12189-4050

8. PERFORMING ORGANIZATION REPORT NUMBER

ARCCB-TR-95028

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U.S. Army ARDEC
Close Combat Armaments Center
Picatinny Arsenal, NJ 07806-5000

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

A systematic study of the rate of convergence for a numerical box-counting and a numerical correlation integral algorithm applied to Euclidean point sets, Koch constructions, and a symmetric chaotic mapping is described. The number of points N_5 required for 5 percent convergence of the box-counting (for $0 \leq q \leq 25$) and correlation integral (for $-25 \leq q \leq 25$) algorithms for the fractal sets studied is determined by the generalized dimension $D(q)$ and is given by $\log_{10}(N_5) \approx 2.54 D(q)-0.11$. Approximately 25 times as many points are required for 1 percent convergence. The box-based correlation integral (BBCI) algorithm employed in the present studies, which is well suited to the analysis of large data sets, is also described.

14. SUBJECT TERMS
Fractal, Multifractal, Correlation Integral, Box-Counting

15. NUMBER OF PAGES
13

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

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INTRODUCTION

A number of algorithms have been devised for the measurement of multifractal parameters. A selection of such algorithms is described in References 1 through 17. The standard algorithms yield the Hentschel-Procaccia (ref 1) fractal dimension $D(q)$, the $f(\alpha)$-spectrum, or other related fractal measures. Optimal utilization of these algorithms for the analysis of experimental data requires an understanding of the practical consequences of representing multifractal sets by imprecise and limited subsets. In the case of machine precision fractal data, it is useful to establish guidelines for the number of points necessary to ensure convergence to sufficient precision. Relevant results have been reported in previous papers.

Reference 2 describes a box-counting algorithm, called the agglomeration box-counting (ABC) algorithm, that is well suited to multifractal analysis of *large* subsets of "pixelized data." Reference 2 discusses the results of ABC convergence studies of two- and three-digit model fractal subsets for $-25 < q < 25$. The rate of convergence for $q \geq 0$ and the nature of the divergence for $q < 0$ is described.

Reference 3 reports the consequences of imprecise data on the effectiveness of the correlation integral method (refs 4-7) and algorithms based on the Badii-Politi principle (refs 8-11). The algorithms are applied to exact (i.e., machine precision) and randomly perturbed *large* (based on the results presented in Reference 2 and some of the results presented in this report) subsets of model multifractal sets in E^2 having $D(q = 0) \leq 2$. It is demonstrated that the correlation integral method can be successfully applied to multifractal data having random errors as large as 1 percent for such sets. The effects of random errors are evident in log-log plots of correlation integral versus radius and an automated procedure for extracting $D(q)$ from such plots is described. It is also demonstrated that reliable generalized dimensions can be obtained from imprecise fractal data by application of a generalized Badii-Politi algorithm for a range of neighbor numbers sufficiently large that the interpoint distances are larger than the random errors in the sets. However, the present authors have yet to devise a technique for selection of appropriate neighbor number in the generalized Badii-Politi algorithm.

In this report, a box-based correlation integral (BBCI) algorithm that is well suited to multifractal analysis of *large* subsets of "pixelized" data is described, and correlation integral and box-counting multifractal analysis of two- and three-digit model subsets for $-25 < q < 25$ for model fractal sets in E^2 having $D(q = 0) \leq 2$ are discussed. Typical results of a convergence study and a table of *sufficient* values of the number of points in the fractal subsets for 1 and 5 percent convergence of the BBCI and the ABC algorithms for subsets of model multifractal sets in E^2 having $D(0) \leq 2$ are presented. The number of points N required for 1 and 5 percent convergence is observed to depend on the generalized dimension $D(q)$ at the q under investigation as

$$\log_{10}(N) \approx 2.54\,D(q) + constant$$

where the constant depends on the degree of convergence.

A convergence study for generalized Badii-Politi algorithms was not performed because there are unresolved questions concerning the interplay of variations of the neighbor number with the size of the fractal subset. Also, a satisfactory (e.g., box-based) form of the generalized Badii-Politi algorithm for *large* data sets is not yet available. However, Kostelich and Swinney (ref 9) suggest that a substantially smaller fractal subset may be sufficient for convergence of the Badii-Politi numerical algorithm.

NUMERICAL TECHNIQUES

The rates of convergence of box-counting and correlation integral multifractal analysis techniques are studied in the context of numerical realizations that are well suited to the analysis of *large* data sets. BBCI and ABC yield D(q) for fractal data represented as elementary hypercube occupation numbers. When precise fractal data are available, the application of these methods entails a loss of information; for example, a point whose coordinates are defined to machine precision merely adds one to the occupation of the appropriate elementary box whose position is defined by sets of three- or four-digit integers.

ABC and BBCI are relatively unaffected by the approximately 0.1 percent imprecision of the boxed data (768x768 boxes) studied here. Reference 3 presented a systematic study of the effects of imprecision on the effectiveness of Badii-Politi and correlation integral fractal analysis algorithms; it is demonstrated that random deviations as large as 1 percent in the positions of the points in a (large enough) subset of a fractal set in E^2 having D(0) < 2.0 has negligible consequences for correlation integral determinations of D(q). Although Reference 2 does not systematically study the effects of imprecision on the effectiveness of box-counting algorithms, it reports that ABC converges to analytic values for three-digit data for q ≥ 0 and that essentially the same results are obtained for two-digit data.

Box-Counting

For q ≠ 1, the box-counting expression for the Hentschel-Procaccia generalized dimension D(q) is determined by

$$(q-1)D(q) = \lim_{E \to 0^-} \left\{ \ln\left(\sum_{i=1}^{N(E)} P_i^q(E) \right) / \ln(E) \right\} \tag{1}$$

where i runs over N(E) occupied hypercubes (boxes) of edge length E and $P_i(E)$ is the probability of finding a point of the fractal set in the ith box. In practice, one deals with finite subsets of the fractal set and determines a numerical approximation to (q-1)D(q) by fitting

$$\ln\left\{ \sum_{i=1}^{N(E)} P_i^q(E) \right\} = (q-1)D(q)\ln(E) + const \tag{2}$$

over an "appropriate" range of E values.

The ABC algorithm employed here uses a selection of E values such that elementary hypercubes are not split. Thus, it is unnecessary to store or search individual point coordinates, enabling relatively simple evaluations of the left-hand side of Eq. (2); computation time is determined solely by the number of occupied elementary boxes and thus is independent of N. The details of ABC are described in Reference 2.

The Correlation Integral Method

For a discrete fractal set, the correlation integral takes the form

$$C(q,E) = \lim_{N\to\infty}\left\{\frac{1}{N}\sum_k^N\left(\frac{1}{N}\sum_j^N H(E-|\vec{x}_k-\vec{x}_j|)\right)^{q-1}\right\}^{\frac{1}{q-1}} \tag{3}$$

where H(x) is the Heaviside function and $\vec{x}_j$ and $\vec{x}_k$ run over N-element fractal subsets. The Hentschel and Procaccia generalized dimension is then given by

$$D(q) = \lim_{E\to 0^+}\frac{\ln(C(q,E))}{\ln(E)} \tag{4}$$

The Box-Based Correlation Integral (BBCI) Algorithm

The BBCI algorithm is a numerical prescription for determining the C(q,E). It has been developed for application to data obtained from image analysis systems, i.e., box occupation data. It is also particularly suitable for the analysis of *large* data sets. (For example, in this study 10^9 point fractal subsets are analyzed.) The latter feature makes BBCI a good choice for convergence studies.

The assignment of the members of machine precision model fractal subsets to boxes in a 768x768 array (used in this investigation) introduces errors of the order of 0.1 percent in the particle positions. However, the results presented in Reference 3 imply that correlation integral analysis yields ≤ 5 percent error values of D(q) for 10^5 point fractal subsets having D(0) ≤ 2.0 and random errors as large as 1 percent.

Since the BBCI algorithm has not been described in the literature before, it is described in detail here. The algorithm is applied as follows:

1. Define an array of hypercubes ("boxes") of edge E_0 appropriate for the given (or anticipated) point set S. Refer to these boxes as elementary hypercubes or elementary boxes.

For experimental data the appropriate choice of E_0 should reflect the inherent uncertainty of the coordinates of the point set within limits set by storage requirements. N.b., storage requirements are determined by the choice of elementary hypercubes rather than the size of the point set; large point sets can be accommodated with relatively modest resources.

2. Compute or measure the occupation numbers n_j for each elementary hypercube.

The total number of points in the subset N can then be expressed as

$$N = \sum_{j}^{elementary\ boxes} n_j \;.$$

3. Define a reference set that comprises a subset of the *occupied* elementary boxes. Let N_{ref} be the number of elementary boxes in the reference set.

4. For each member of the reference set, define a set of hypercube edge lengths,

$$E = (2n+1)E_0, \; where \; n = 0,1,2,...n_M \;.$$

5. For each q of interest:

a. Compute the box-based generalized correlation integrals

$$C(q,E,E_0) \;=\; \left\{ \frac{1}{N_R} \sum_{r}^{N_{ref}} \left(\frac{1}{N} \sum_{j}^{elementary\ boxes} n_j G(E,E_0,\vec{x}_r - \vec{x}_j) \right)^{q-1} \right\}^{\frac{1}{q-1}} \tag{5}$$

for the E values defined in step 4, where r runs over the reference set, j runs over all elementary boxes,

$$N_R = \sum_{k}^{N_{ref}} n_k$$

and

$$G(E,E_0,\vec{x}) \;=\; \begin{cases} 1, & x_j \leq (E-E_0)/2, \; for \; all \; components \; of \; x \\ 0, & otherwise \end{cases} \tag{6}$$

selects the elementary boxes contained in larger hypercubes of edge E, centered on x_r. The vectors x_r and x_j point to elementary hypercubes rather than members of the point set.

BBCI reduces to the standard (finite N) generalized correlation integral (refs 4-7) in the $E_0 \to 0$ limit if the coordinates of the members of the multifractal subset in question are precisely known, i.e.,

$$\lim_{E_0 \to 0^+} C(q,E,E_0) \to C(q,E)$$

In practice, computation was slowed and convergence was not improved by employing approximate hyperspheres rather than hypercubes.

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b. Obtain D(q) by linear regression on

$$\ln(C(q,E,E_0)) = const + D(q)\ln(E) \qquad (7)$$

for $E \in \{(2n+1)E_0 | n = n_{min},...,n_{max}\}$ *with* $n_{min} > 0$.

RESULTS AND DISCUSSION

The ABC and the BBCI methods have been developed for the analysis of "pixelized" image acquisition system data. The algorithms are particularly well suited to the analysis of large data sets and are therefore appropriate for convergence studies. The reported results are obtained using a 768x768 array of elementary boxes. Results are not given for other size arrays. However, as discussed in Reference 2, as the number of elementary boxes increases, the accuracy of the converged values tends to improve but the number of points required for convergence increases. The BBCI convergence results for the representations of the fractal sets reported here are consistent with the *standard* correlation integral results reported in Reference 3 for $N \approx 2 \times 10^5$ machine precision (and also for 0.1 and 1.0 percent randomly perturbed) representations of the same fractal sets. ABC results are consistent with results obtained using standard "sorting" box-counting algorithms at relatively small values of N for the same model sets.

CPU Time

CPU time is proportional to N^2 for the standard correlation integral method and is proportional to $N \ln(N)$ for standard box-counting algorithms. ABC's CPU time depends only on the number of elementary boxes and is essentially independent of N. BBCI's CPU time is a function of the size of the reference set, the number of occupied elementary boxes, and the number of hypercube edge lengths employed. The number of occupied elementary boxes increases with N over a substantial range of N before saturating. The results were obtained using all occupied boxes for the reference set and 40 hypercube edge lengths. Essentially equivalent results were obtained with one-fourth of the CPU time when 25 percent of the occupied elementary boxes comprise the reference set. BBCI's CPU time is less than ABC's CPU time for small N but substantially greater at large N. CPU times are essentially the same for N between 10^3 to 10^4. For example, execution times on an SGI IRIS 4D/33-MHz workstation are 15, 25, and 30 seconds using ABC and 35, 160, and 320 seconds using BBCI for N = 15,000, 160,000, and 2×10^9 point representations of split snowflake halls, respectively.

Convergence

Convergence properties of BBCI and ABC have been determined for randomly oriented Euclidean sets, Koch asymmetric (ref 18) [0.4,0.2] and symmetric triadic snowflakes, split snowflake halls (ref 18), the 13-element generator Koch construction (ref 18), and the attractor for the sixfold (D6) symmetric chaotic mapping in Figure 3 of Reference 19. BBCI (ABC) converged within 1 percent (4 percent) of analytic values in all cases studied. The accuracy may be improved by employing a procedure similar to that described in the Appendix of Reference 3.

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Figure 1 shows typical results of applying ABC and BBCI to the model fractal sets at $q \geq 0$. BBCI results are similar at $q < 0$, where ABC usually diverges. The small $\log_{10}(N)$ variations are not always as smooth as shown and the converged value not as well defined. The graph displays measured $D(q)$ versus $\log_{10}(N)$ for the 13-element Koch construction at $q = 5$. The measured $D(5)$ values are represented by open circles that are connected by lines. The horizontal line is the analytic value of $D(5)$ for the construction. The BBCI (ABC) converged value is within 0.3 percent (2 percent) of the analytic value and generally BBCI (ABC) converged within 1 percent (4 percent) of the analytic results.

Table 1 summarizes the results of the convergence studies for the fractal sets. N_1 (N_5) is the number of points *sufficient* for the algorithm to yield values converged within 1 percent (5 percent) of values obtained for the largest N studied. The N_1 values are generally less well defined than the N_5 values because the convergence curves tend to flatten out as N increases. The data of Figure 1 illustrate the problem implicit in using a sufficiency criterion; the BBCI N_5 value corresponds to an actual discrepancy of only 1.1 percent. The N_5 discrepancies are larger than 1.5 percent in the rest of Table 1. The values given with asterisks correspond to cases where convergence is not clearly established at the largest N (generally of the order of 10^{11}) and probably overestimate N_1.

Figures 2 and 3 are semi-log plots of the N_5 and N_1 data of Table 1 against the analytic values of $D(q)$ for the model fractal sets. The line in Figure 2 is obtained by least squares fitting of the 5 percent BBCI data to the form,

$$\log_{10}(N_5) = a D(q) + b \qquad (8)$$

with $a = 2.54 \pm 0.21$ and $b = -0.11 \pm 0.34$.

The line in Figure 3 is not fit to the 1 percent BBCI data, but it is drawn parallel to the least squares line through the 5 percent BBCI data. The constant b is adjusted ($a = 2.54, b = 1.20$) to approximate the asterisk-free 1 percent BBCI results given in Table 1. ABC and BBCI converge at about the same rate for $q \geq 0$ and require approximately 25 times more points for 1 percent convergence than for 5 percent convergence.

The results presented in Table 1 and Figures 2 and 3 provide guidelines for the application of numerical fractal analysis algorithms to multifractal subsets and support the contention (refs 20,21) that the number of points necessary for a given degree of convergence increases exponentially with D.

Table 1. Sufficient values of N to yield 1% (N_1) and 5% (N_5) convergence.
Box-counting (ABC) does not converge for q<0 as a rule.
Values marked with asterisks are rough estimates.

	q =	-25	-5	0	5	25
ABC	N_1			1.2E4	1.2E4	3.1E3
	N_5			3.1E3	770	770
[0.4,0.2], Asymmetric Snowflake D(0)=1.16.						
BBCI	N_1	7.9E4	5.2E4	1.2E4	770	3.1E3
	N_5	3.1E3	3.1E3	770	190	770
ABC	N_1			1.2E4	1.2E4	5.2E4
	N_5			3.1E3	3.1E3	3.1E3
Triadic Snowflake, D(q) = 1.26 for all q.						
BBCI	N_1	1.2E4	1.2E4	1.2E4	1.2E4	5.0E4
	N_5	770	770	770	770	770
ABC	N_1			1.8E6	1.6E5	1.6E5
	N_5			1.6E5	1.5E4	1.5E4
Split Snowflake Halls, D(0) = 1.86.						
BBCI	N_1	1.0E8*	2.0E7	1.6E5	1.6E5	1.6E5
	N_5	1.6E5	1.6E5	1.5E4	1.5E4	1.5E4
ABC	N_1			3.7E5	3.7E5	3.7E5
	N_5			2.9E4	2.9E4	2.9E4
13-Element Generator, D(0) = 2.0.						
BBCI	N_1	1.0E8*	4.8E6	2.9E4	3.7E5	3.7E5
	N_5	4.8E6	3.7E5	2.9E4	2.9E4	2.2E3
ABC	N_1	1.0E8*	1.0E8*	1.0E6	1.0E5	1.0E6
	N_5	1.0E8*	1.0E8*	1.0E5	1.0E5	1.0E5
Sixfold Symmetric Mapping, D(0)=2.0.						
BBCI	N_1	1.0E7	1.0E7	1.0E6	1.0E5	1.0E6
	N_5	1.0E7	1.0E6	1.0E5	1.0E5	1.0E5

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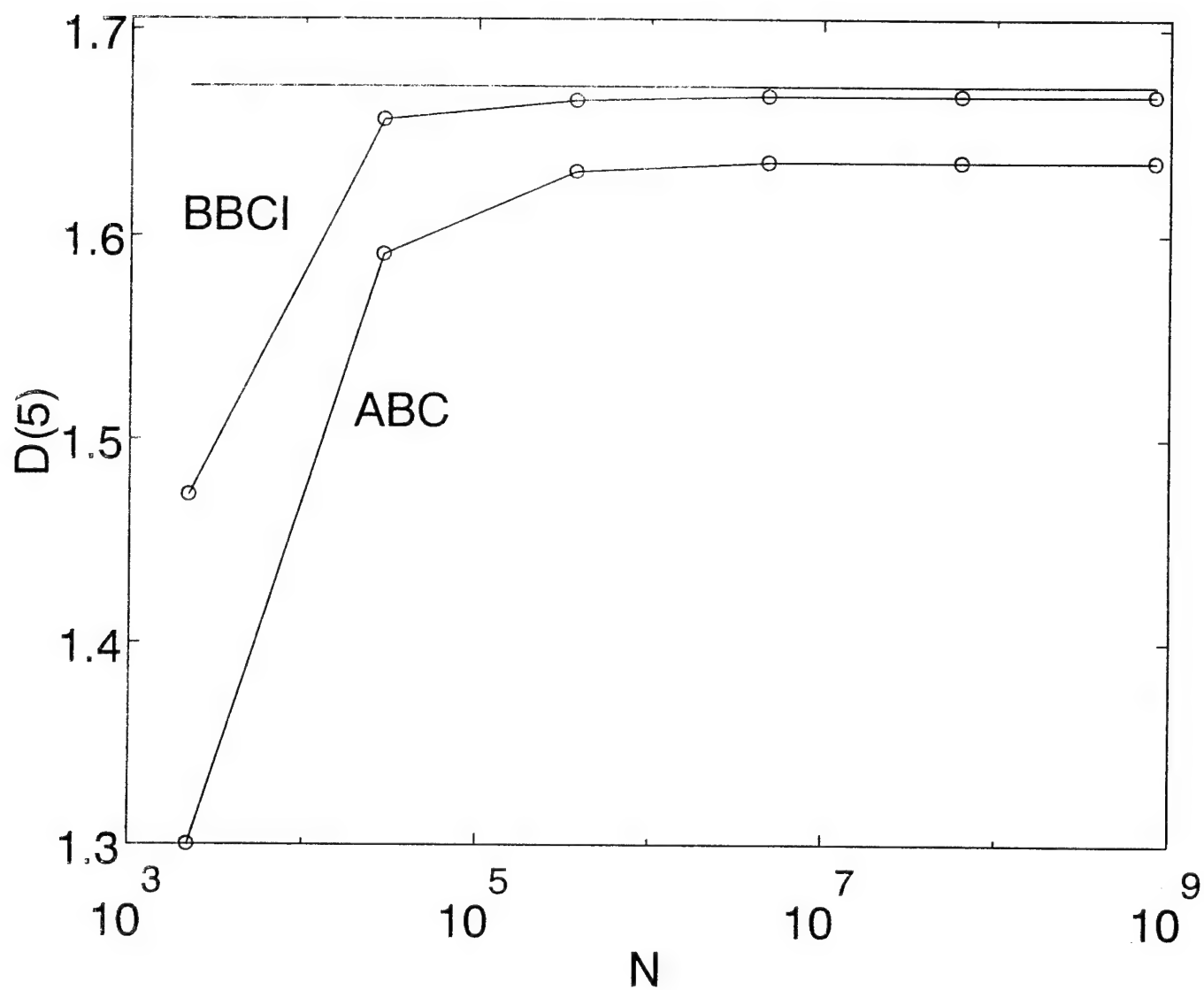
Figure 1. Hentschel and Procaccia generalized dimension D(5) versus the logarithm of the number of points in the fractal subset for the 13-element generator (ref 18) construction.

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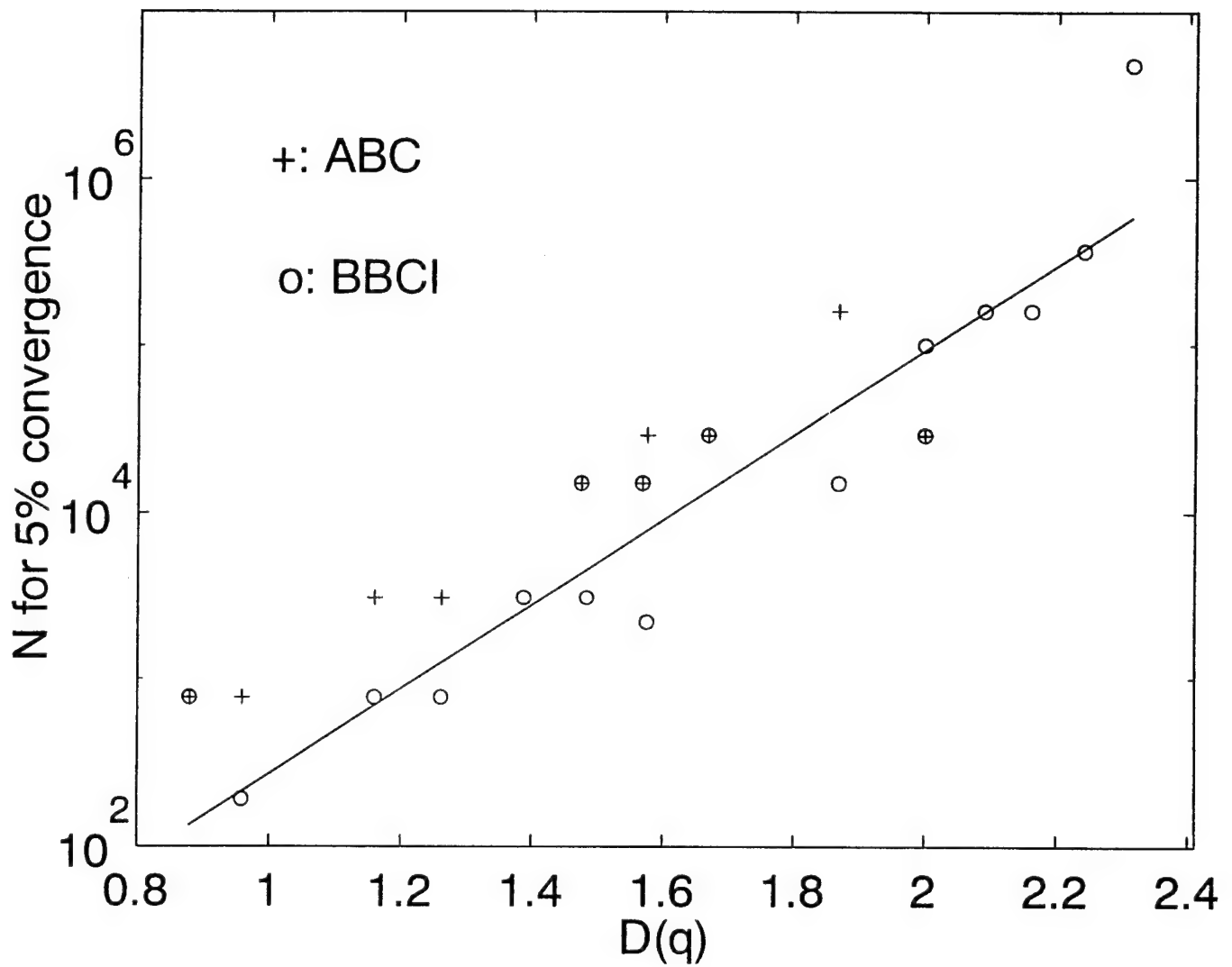Figure 2. Logarithm of the number of points for 5 percent convergence versus the Hentschel and Procaccia generalized dimension D(q). The solid line is given by: $\log_{10}(N_5) = 2.54\,D(q) - 0.11$.
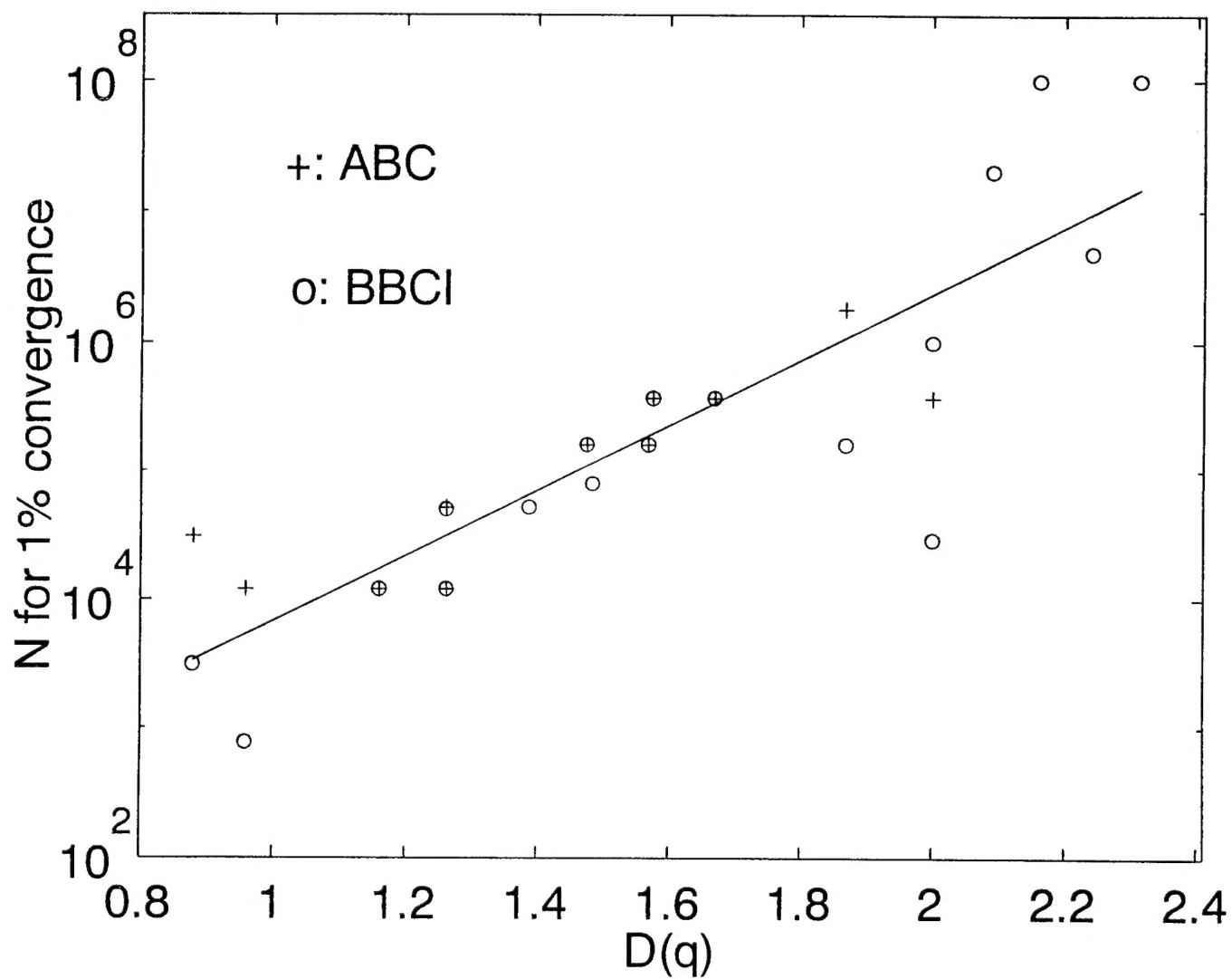
Figure 3. Logarithm of the number of points for 1 percent convergence versus the Hentschel and Procaccia generalized dimension D(q). The solid line is given by: $\log_{10}(N_1) = 2.54\,D(q) + 1.20$.

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